AP Calc BC Parametric Test Name: Block: Seat:

2 points each

1. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$ for $-3 \le t \le 3$ is a line segment with slope
   (a) $\frac{3}{5}$
   (b) $\frac{5}{3}$
   (c) 3
   (d) 5
   (e) 13
   (f) None of these

A

$\frac{dy}{dt}$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3}{5}$

2. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$ is given by
   (a) $\int_0^1 \sqrt{t^2 + 1}\, dt$
   (b) $\int_0^1 \sqrt{t^2 + t}\, dt$
   (c) $\int_0^1 \sqrt{t^4 + t^2}\, dt$
   (d) $\frac{1}{2}\int_0^1 \sqrt{4 + t^2}\, dt$
   (e) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$
   (f) None of these

C

$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dx}{dt} = t^2 \qquad \frac{dy}{dt} = t$

$\int_0^1 \sqrt{t^4 + t^2}\, dt$

2 points each

1. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$ for $-3 \leq t \leq 3$ is a line segment with slope

   (a) $\frac{3}{5}$

   (b) $\frac{5}{3}$

   (c) 3

   (d) 5

   (e) 13

   (f) None of these

$y = 3t \Rightarrow t = \frac{y}{3}$

$x = 5\left(\frac{y}{3}\right) + 2$

$y = \frac{3}{5}(x - 2)$

2. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \leq t \leq 1$ is given by

   (a) $\int_0^1 \sqrt{t^2 + 1}\, dt$

   (b) $\int_0^1 \sqrt{t^2 + t}\, dt$

   (c) $\int_0^1 \sqrt{t^4 + t^2}\, dt$

   (d) $\frac{1}{2}\int_0^1 \sqrt{4 + t^2}\, dt$

   (e) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$

   (f) None of these

$x' = t^2 \quad y' = t$

$\int_0^1 \sqrt{t^4 + t^2}\, dt$

3. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

   (a) $-\frac{5}{2}$
   (b) $-\frac{6}{5}$
   (c) 0
   (d) $\frac{4}{5}$
   (e) $\frac{6}{5}$
   (f) None of these

B

x	y
$\frac{dx}{dt}$	$\frac{dy}{dt}$

$2(5) = 10$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$(2)(3) + (5)\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-6}{5}$$

slope at $(2,5)$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-5}{2}$$

$$\frac{dx}{dt} = 3 \cdot \frac{-2}{5}$$

4. For $0 \le t \le 18$ an object travels along an elliptical path given by the parametric equations $x = 3\sin t$ and $y = 4\cos t$. At the point where $t = 18$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

   (a) $-\frac{4}{3}$
   (b) $-\frac{3}{4}$
   (c) $-\frac{4\tan 18}{3}$
   (d) $-\frac{4}{3\tan 18}$
   (e) $-\frac{3}{4\tan 18}$
   (f) None of these

C

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -4\sin t$$

$$\left.\frac{dy}{dx}\right|_{t=18} = \left.\frac{-4\sin t}{3\cos t}\right|_{t=18} = \left.-\frac{4}{3}\tan t\right|_{t=18}$$

5. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t does the path have a vertical asymptote?
   (a) -1 only
   (b) 0 only
   (c) 2 only
   (d) -1 and 2 only
   (e) -1, 0, and 2
   (f) None of these

when $\frac{dx}{dt} = 0$ & $\frac{dy}{dt} \neq 0$

$3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0$ & $t = 2$

$\frac{dy}{dt} = 6t^2 - 6t - 12$

$\frac{dy}{dt} = 6(t^2 - t - 2) = 6(x-2)(x+1) = 0$

$t = 2, -1$

6. A curve C is defined by the parametric equations $x = t^2 - 8t + 12$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-4, 64)$?
   (a) $x = -4$
   (b) $x = 4$
   (c) $y = 64$
   (d) $y = -\frac{27}{10}(x+4) + 64$
   (e) $y = 48(x+4) + 64$
   (f) None of these

$t^2 - 8t + 12 = -4$

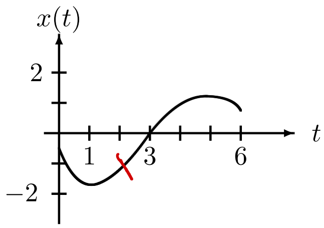
$t^2 - 8t + 16 = 0$

$(t-4)^2 = 0$

at $t = 4$ $x = -4$, $y = 64$

$\frac{dy}{dx} = \frac{3t^2}{2t - 8}\Big|_{t=4} = \frac{3(16)}{8-8}$

so vert asymptote



x: $-$ | $+$ at 3

$v(t) = x'$: $-$ | 1 | $+$ | 5

$v'(t) = x''$: $+$ | 2 | $-$

7. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

   (a) $0 < t < 2$
   (b) $1 < t < 5$
   (c) $2 < t < 6$
   (d) $3 < t < 5$ only
   (e) $1 < t < 2$ and $5 < t < 6$

vel. inc $\Rightarrow$ accel is pos (when pos curve is concave up)

8. Consider the curve given by $x = 2\sin t$ and $y = 4\cos^2 t$ for $0 \le t \le \pi$. Find $\dfrac{d^2y}{dx^2}$

   (a) -2
   (b) $\dfrac{-2}{\tan t}$
   (c) $-2t$
   (d) $-4\sin t$
   (e) None of these

$$\frac{dy}{dx} = \frac{4(-2\cos t \sin t)}{2\cos t} = -4\sin t$$

$$\frac{d^2y}{dx^2} = \frac{-4\cos t}{2\cos t} = -2$$

2008 MC #28

$\frac{dy}{dx} = 2x - 1$

9. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

   (a) $\frac{2}{3}$

   (b) $\frac{2\sqrt{10}}{3}$

   (c) 3

   (d) 6

   (e) $6\sqrt{10}$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (2\sqrt{10})^2$

$\left(\frac{dy}{dt}\right)^2 = 40 - \left(\frac{dx}{dt}\right)^2$

$2x - 1 = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

$\frac{dy}{dt} = (2x-1)\left(\frac{dx}{dt}\right)$

$\left(\frac{dy}{dt}\right)^2 = (2x-1)^2\left(\frac{dx}{dt}\right)^2$

$40 - \left(\frac{dx}{dt}\right)^2 = (2x-1)^2\left(\frac{dx}{dt}\right)^2 \Big|_{(x=2)}$

$40 - \left(\frac{dx}{dt}\right)^2 = 9\left(\frac{dx}{dt}\right)^2$

$40 = 10\left(\frac{dx}{dt}\right)^2$

$\frac{dx}{dt}^2 = \frac{40}{10} = 4$

$\frac{dy}{dt} = \sqrt{40 - 4} = \sqrt{36} = 6$

10. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

   (a) 2.909

   (b) 3.062

   (c) 6.884

   (d) 9.016

   (e) 47.393

   (f) None of these

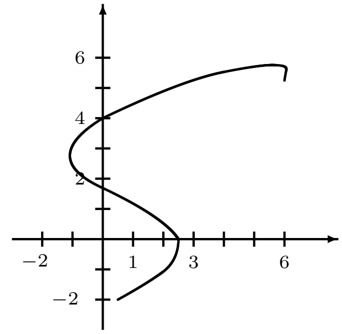
$\sqrt{(2t)^2 + [4\cos(4t)]^2}\,\Big|_{t=3}$

6.884285784

2013 Practice Exam BC 2

11. A planetary rover travels on a flat surface. The path of the rover for the time interval $0 \le t \le 2$ hours is shown in the rectangular coordinate system below. The rover starts at the point with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position of the rover change at rates given by

$$x'(t) = -12\sin(2t^2)$$
$$y'(t) = 10\cos(1 + \sqrt{t}),$$



where $x(t)$ and $y(t)$ are measured in meters and t is measured in hours.

(a) (2 points) Find the speed of the rover at time $t = 1$.

$$\sqrt{[x'(t)]^2 + [y'(t)]^2}\Big|_{t=1} = 11.678 \text{ m/hr}$$

at time $t = 1$ the speed was 11.678 meters per hour

(b) (3 points) Find the total distance that the rover travels over the time interval $0 \le t \le 1$.

$$\int_0^1 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

Traveled 6.704 meters in the first hour

(c) (2 points) Find the y-coordinate of the position of the rover at time $t = 1$.

$$y(0) + \int_0^1 y'(t)\,dt$$

$$5 + \int_0^1 y'(t)\,dt = 4.057 \quad (\text{or } 4.056)$$

(d) (2 points) The rover receives a signal at each point where the line tangent to its path has slope $\frac{1}{2}$. At what times t, for $0 \le t \le 2$, does the rover receive a signal?

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10\cos(1-\sqrt{t})}{-12\sin(2t^2)} = \frac{1}{2}$$

$t = 1.072$ hours

2004 Form B BC 1

12. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$

(a) (2 points) Find the speed of the particle at time $t = 0$

$$\sqrt{(x'(0))^2 + (y'(0))^2}$$

$$= \left.\sqrt{t^4 + 9 + (2e^t + 5e^{-t})^2}\right|_{t=0}$$

$$= \sqrt{9 + \left(2 + \frac{5}{1}\right)^2}$$

$$= \sqrt{58}$$

(b) (2 points) Find an equation of the line tangent to the path of the particle at time $t = 0$

$$\frac{dy}{dt} = \left.\frac{2e^t + 5e^{-t}}{\sqrt{t^4 + 9}}\right|_{t=0} = \frac{2 + \frac{5}{1}}{3} = \frac{7}{3}$$

$$y - 1 = \frac{7}{3}(x - 4)$$

(c) (3 points) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$

$$\int_0^3 \sqrt{t^4 + 9 + (2e^t + 5e^{-t})^2}\, dt$$

45.226

or

45.227

(d) (2 points) Find the x-coordinate of the position of the particle at time $t = 3$

$$x(0) + \int_0^3 x'(t)\, dt$$

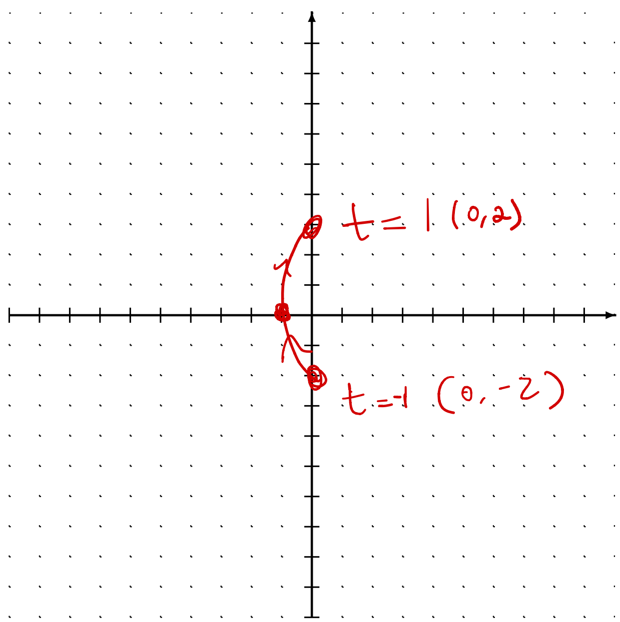
$$4 + 13.930 = 17.930$$

or

$$4 + 13.931 = 17.931$$

Gary Taylor 229 #20 4.591

13. Find the length of the arc between the two y-intercepts of the parametric curve $x(t) = t^2 - 1$ and $y(t) = 2t$. Be sure to show a directed graph, an integral, and a conclusion.



y intercepts (when $x = 0$)

$t^2 - 1 = 0$

$t = \pm 1$

$\left(\frac{y}{2}\right)^2 - 1 = x$

$\frac{y^2}{4} = x + 1$

$y^2 = 4x + 4$

$y = \pm\sqrt{4x+4}$

$$\int_{-1}^{1} \sqrt{(2t)^2 + (2)^2}\, dt$$

$$\int_{-1}^{1} \sqrt{4t^2 + 4}\, dt$$

$$2\int_{-1}^{1} \sqrt{t^2+1}\, dt = 4.591$$

the length of the arcle for $t = -1$ to 1 is 4.591